Leontief versus Ghoshian Price and Quantity Models*

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I. Introduction

The classical, "demand-driven" input-output model dates back to Leontief [24; 25]. Its economically complete opposite, the so-called "supply-driven" input-output model was developed almost forty years ago by Ghosh [16], whereas its first application came more than ten years later by Augustinovics [1], followed by Giarratani [18]. The first theoretical reservations were formulated another ten years later by Giarratani [19] and Oosterhaven [31, 140–1].

Despite these reservations, the model came more or less *en vogue* in the 1980s. Several applications appeared; at best paying only lip service to this earlier critique [10; 5]. Moreover, without much reservations the model also entered the major input-output textbook [28, 317–22], while uncritical generalizations and reiterations appeared in the theoretical literature [8; 3; 9]. Then a heavy discussion about the economic underpinning of the supply-driven quantity model was initiated by Oosterhaven [33; 36; 21; 34]. With that debate the enthusiasm to perform downright applications of the supply-driven model seems to have diminished.

Literally in the footnotes of that debate, the (non-)existence, the mathematics and the possible economic interpretation of the dual, price version of the supply-driven model was discussed [33; 34]. Independently of this discussion, Davar [9] presented the dual model in extension, but did not discuss the economic plausibility of the underlying economic assumptions nor their implications. In view of the misinterpretations and misuse of the quantity version of the supply-driven model, this contribution aims at an early evaluation of its price version that is as complete and as balanced as possible.

It is well known that the dual version of the regular input-output model is used to simulate cost-push inflationary processes [26, 188–201; 7, 246]. Other applications relate to, for instance, the price effects of pollution abatement [17], effects of rising energy cost [29], and energy price effects in multiregional or in extended interregional models [35; 32]. Of course, this dual, price version of the Leontief model is based on the same stringent, standard assumptions of the quantity version and on some heavy, additional assumptions that are made for the price version [37].

Naturally, the potential use of the dual of the supply-driven model lies in the simulation of demand-pull inflationary processes as opposed to the cost-push applications of the dual Leontief model. Hence, part of our evaluation will be directed toward the question whether such applications are justified from an economic point of view or not.

Our discussion will start in the next section with a brief summary of the Leontief quantity and

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price models and their mutual relationship. Next, both Ghoshian models will be presented, followed by a discussion of their economic plausibility. Our essentially negative verdict is combined with some concluding remarks in the last section.

II. The Leontief Quantity and Price Model in Brief

For both the Leontief and the Ghoshian input-output models we assume a closed economy with I intermediate sectors (i.e., industries), Q final sectors (i.e., final demand categories) and P primary sectors (i.e., value added categories). Both models start from the same base year input-output table on which their accounting identities are based.

The mathematics of the well known *Leontief model* may be summarized in two sets of equations. First, the base year accounting identities for total output, across the rows of the i-o table, with all (index)prices standardized at one:

$$\mathbf{x} = \mathbf{Zi} + \mathbf{Yi} = \mathbf{Zi} + \mathbf{y} \tag{1}$$

and, second, the behavioral relations with fixed intermediate and primary input coefficients, derived from the columns of the i-o table:

$$\mathbf{Z} = \mathbf{A\hat{x}} \quad \text{and} \quad \mathbf{V} = \mathbf{C\hat{x}} \tag{2}$$

where:

$\mathbf{x}$ = I-vector with total output (= total input) per industry,
$\mathbf{Z}$ = $I \times I$-matrix with intermediate outputs (= intermediate inputs),
$\mathbf{Y}$ = $I \times Q$-matrix with final outputs,
$\mathbf{V}$ = $P \times I$-matrix with primary inputs,
$\mathbf{i}$ = vector of the appropriate size with ones, i.e., a summation vector,
$\hat{}$ = diagonal matrix made from the corresponding vector,
$\mathbf{A}$ = $I \times I$-matrix with fixed intermediate input coefficients,
$\mathbf{C}$ = $P \times I$-matrix with fixed primary input coefficients.

The demand-driven model (1)–(2) with its *exogenous final output* has the following well known solutions for endogenous total output:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} = \mathbf{Ly} \tag{3}$$

and *endogenous* intermediate and *primary inputs:*

$$\mathbf{Z} = \mathbf{A}\,\widehat{\mathbf{Ly}} \quad \text{and} \quad \mathbf{V} = \mathbf{C}\,\widehat{\mathbf{Ly}} \tag{4}$$

where: $\mathbf{L} = I \times I$ so-called Leontief-inverse.

The price version of the Leontief model is almost equally well known [7; 37]. We will call it the *cost-push i-o price model* in order to make a clear distinction with its demand-pull opposite.[1] The

1. Davar [9] used the label "demand price model" for the Ghoshian price model, as he chooses the somewhat unusual label 'supply price model' for the Leontief price model. To be more precise economically and to prevent misunderstanding, I will use the labels "cost-push price model" and "demand-pull price model."

cost-push price model is based on the following accounting identity for the values of sectoral total inputs (i.e., total cost):

$$\mathbf{p}'\hat{\mathbf{x}} = \mathbf{p}'\mathbf{Z} + \mathbf{p}_\mathrm{v}'\mathbf{V} \tag{5}$$

where:

$\mathbf{p}$ = *I*-vector with (index)prices for total sectoral output,
$\mathbf{p}_\mathrm{v}'$ = *P*-vector with (index)prices for primary inputs per category.

Next, the price version, just like the quantity version, assumes fixed input coefficients. Substitution of (2) in (5) and post-multiplication by $\hat{\mathbf{x}}^{-1}$ gives the unit costs per sector (which under perfect competition equals the prices for total output) as the weighted average of the prices for the intermediate and primary inputs:

$$\mathbf{p}' = \mathbf{p}'\mathbf{A} + \mathbf{p}_\mathrm{v}'\mathbf{C}. \tag{6}$$

Note that the sum of the weights for each industry equals one: $\mathbf{i}'\mathbf{A} + \mathbf{i}'\mathbf{C} = \mathbf{i}'$, as the input accounting identities in (9) are measured in unit price-indices of the base-year. Hence, for this base-year: $\mathbf{p}' = \mathbf{i}' = \mathbf{p}_\mathrm{v}'$.

The essence of the Leontief price model lies in the additional assumptions regarding the causal relationships between these prices. The *primary* input prices (homogeneous per row) are assumed to be *exogenous*, whereas the *I* prices for the single, homogeneous outputs are determined by the solution of the model:

$$\mathbf{p}' = \mathbf{p}_\mathrm{v}'\mathbf{C}(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{p}_\mathrm{v}'\mathbf{CL}. \tag{7}$$

The economic interpretation of (7) is simple. Primary input prices $\mathbf{p}_\mathrm{v}'$ may change exogenously. The size of the direct effect of such changes on endogenous unit cost (= output prices $\mathbf{p}'$) is determined by the fixed primary input coefficients $\mathbf{C}$. Next, the prices of intermediate inputs rise as firms compensate their unit cost increases in their output prices. The subsequent endogenous increases in intermediate input prices again cause output prices to rise, the cumulative effect of this cost-push process is described by the Leontief-inverse in (7).

To further clarify the relation between the quantity and the price version of the Leontief model, (7) is postmultiplied by $\mathbf{y}$, which gives:

$$\mathbf{p}'\mathbf{y} = \mathbf{p}_\mathrm{v}'\mathbf{CLy} = \mathbf{p}_\mathrm{v}'(\mathbf{Vi}). \tag{8}$$

On the one hand, (8) expresses the equality in value between total final output and total primary inputs, but, on the other hand, (8) emphasises the independency (or better duality) of the quantity and price model. Although their solutions are linked through (8), their variables remain independently determined, namely, $\mathbf{v}$ by $\mathbf{y}$ in (4), and $\mathbf{p}'$ by $\mathbf{p}_\mathrm{v}'$ in (7). Consequently, each market in functions as in Figure 1. Prices and quantities move independently. Supply is perfectly price elastic and demand is perfectly price *in*elastic.

From this it is clear why (7) is used to simulate cost-push inflationary processes. Starting with primary input price increases (e.g., of wages, capital costs, indirect taxes/subsidies or imports) via unit cost and intermediate input price increases, the inflationary process ends with final output price increases. Crucial is the assumption that each price increase is entirely passed on to

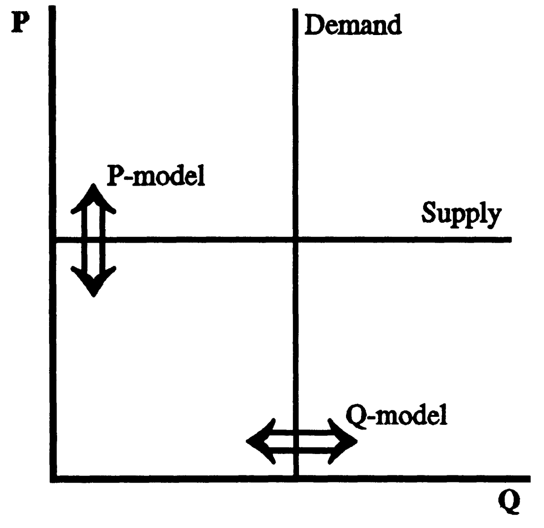

Figure 1. Markets in the Leontief Input-Output Models

pure price-taking purchasers whose demand does not react at all. Clearly, reality is a bit more complicated than that.

III. The Ghoshian Quantity and Price Model

The mathematics of the *Ghoshian model* is equally simple, but opposite. In stead of accounting identities for total output, it starts with base year accounting identities for total *input,* across the columns of the i-o table, again with all (index)prices set equal to one:

$$\mathbf{x}' = \mathbf{i}'\mathbf{Z} + \mathbf{i}'\mathbf{V} = \mathbf{i}'\mathbf{Z} + \mathbf{v}'. \tag{9}$$

Note that (5) gives an identity for the values of the heterogeneous inputs per sector, whereas (9) gives an identity for the quantities of the single homogeneous input per sector.

Next, instead of the fixed intermediate and primary input coefficients of the Leontief model, the Ghoshian model uses fixed intermediate and final *output* coefficients, derived from the rows of the i-o table:

$$\mathbf{Z} = \hat{\mathbf{B}} \quad \text{and} \quad \mathbf{Y} = \hat{\mathbf{x}}\mathbf{D} \tag{10}$$

where:

$\mathbf{B} = I \times I$-matrix of fixed intermediate output coefficients,
$\mathbf{D} = I \times Q$-matrix of fixed final output coefficients.

Furthermore, in stead of exogenous final outputs, the Ghoshian model (9)–(10) has *exogenous primary inputs,* and in stead of the Leontief solution for total outputs, the Ghoshian model produces a solution for endogenous total inputs:

$$\mathbf{x}' = \mathbf{v}'(\mathbf{I} - \mathbf{B})^{-1} = \mathbf{v}'\mathbf{G}. \tag{11}$$

Finally, opposed to the Leontief solution for endogenous intermediate and primary inputs, total inputs from (11) are used to derive the subsequent Ghoshian solutions for *endogenous* intermediate and *final outputs:*

$$\mathbf{Z} = \mathbf{v}'\mathbf{G}\mathbf{B} \quad \text{and} \quad \mathbf{Y} = \mathbf{v}'\mathbf{G}\mathbf{D} \tag{12}$$

where: $\mathbf{G} = I \times I$ so-called Ghoshian inverse.

The price version of the supply-driven input-output model, which we will call the *demand-pull i-o price model,* is not based on the accounting identity for the value of total input (5), but on that for total output (i.e., total revenue):

$$\hat{\mathbf{x}}\mathbf{p} = \mathbf{Z}\mathbf{p} + \mathbf{Y}\mathbf{p}_\mathbf{y} \tag{13}$$

where:

$\mathbf{p}$ = I-vector of (index)prices for total sectoral input,
$\mathbf{p}_\mathbf{y}$ = Q-vector of (index)prices for final output per category.

Note that equation (13) gives an identity for the values of heterogeneous outputs, whereas identity (1) relates to the quantities of a single homogeneous output. As opposed to the cost-push model where $\mathbf{p}$ refers to the price for each sector's single homogeneous output, $\mathbf{p}$ in equation (13) relates to the price for each sector's single homogeneous *input.* Furthermore, as opposed to the cost-push model where primary input was homogeneous across rows, here final output is homogeneous across each column of the input-output table. These are of course two rather severe assumptions.

Next, the fixed output coefficient assumption (10) is added again. Substitution of (10) in (13) and premultiplication by $\hat{\mathbf{x}}^{-1}$ gives the unit revenues per sector (which under perfect competition equals the price of each sector's single input) as the weighted average of that sector's intermediate and final output prices:

$$\mathbf{p} = \mathbf{B}\mathbf{p} + \mathbf{D}\mathbf{p}_\mathbf{y}. \tag{14}$$

Note that the sum of the output weights per industry is equal to one: $\mathbf{Bi} + \mathbf{Di} = \mathbf{i}$, as accounting identity (1) is measured for the base-year in unit price indices. Hence, for the base-year we have: $\mathbf{p} = \mathbf{i} = \mathbf{p}_\mathbf{y}$.

The direction of causality in this demand-pull price model runs opposite to that of the cost-push price model. The single price for each (homogeneous) column with final outputs is assumed exogenous, whereas tne prices for primary inputs per sector are now endogenous. This gives the following solution for the endogenous price of total (intermediate and primary) input per sector:

$$\mathbf{p} = (\mathbf{I} - \mathbf{B})^{-1}\mathbf{D}\mathbf{p}_\mathbf{y} = \mathbf{G}\mathbf{D}\mathbf{p}_\mathbf{y}. \tag{15}$$

The only possible economic interpretation of (15) justifies the label "demand-pull i-o price model" as it describes the cumulative effects of changes in final output prices on the prices of (primary) inputs, such as labor and the use of capital.

In this interpretation, the direct effect of an exogenous change in the price of a final output on unit revenues per sector is given by the importance of that category of final output for that sector (i.e., by $\mathbf{D}$). Under perfect competition, this increase in unit revenues is entirely passed

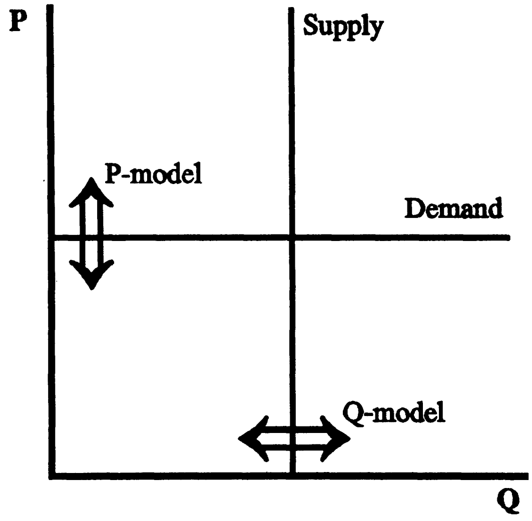

Figure 2. Markets in the Ghoshian Input-Output Models

on to the price for the single homogeneous input. As the price of total input per sector thus increases, the prices of all intermediate inputs must also increase, not row-wise as in the cost-push model but column-wise. This means that in each column of the input-output table prices of all intermediate inputs change with that sector's increase in total input price. Given the fixed intermediate output coefficients, the importance for these intermediate output price changes for sectoral unit revenue is determined in each round of indirect price effects by **B**. The cumulative effect of these intermediate price increases is described by the Ghoshian inverse in equation (15).

Premultiplication of equation (15) by $\mathbf{v}'$ clarifies the duality between the price and quantity versions of the Ghoshian model:

$$\mathbf{v}'\mathbf{p} = \mathbf{v}'\mathbf{G}\mathbf{D}\mathbf{p}_y = \mathbf{x}'\mathbf{D}\mathbf{p}_y = (\mathbf{i}'\mathbf{Y})\mathbf{p}_y. \tag{16}$$

Obviously, in the Ghoshian models the equality between the value of total primary inputs and total final outputs is preserved, as in the Leontief models. More importantly, (16) shows how the solutions of the price and quantity models are linked in value terms, but are again determined independently. Changes in primary inputs $\mathbf{v}'$ determine the changes in final output $\mathbf{y}$ in the Ghoshian quantity model (12), whereas the causality between the prices in the demand-pull model (15) runs opposite. Final output prices $\mathbf{p}_y$ (per homogeneous column, such as consumption, investments or exports) determine the similar price $\mathbf{p}$ of all inputs per sector.

Figure 2 shows this independent nature of the price and quantity version of the Ghoshian model, now from the perspective of the functioning of individual markets. Purchase prices move independently and are set by the buyers of output. Hence, firms maximize their revenues by holding their output coefficients constant, passing any increase in output prices (i.e., unit revenues) on to their suppliers, from which they accept any quantity supplied, as all (intermediate and primary) inputs are perfect substitutes for each other!

IV. A Comparison of Both Sets of Models

The interesting aspect about these two opposite models is the empirical evidence. The research into the stability of the input coefficients vis-à-vis the stability of the output coefficients appears to be rather inconclusive [1; 15; 20; 2]. Helmstädter and Richtering [22], for West-Germany 1960–1975, even report output coefficients to be more stable, and consequently find smaller prediction errors for the supply-driven model.

This inconclusive result is not too surprising if one acknowledges that the two sets of most crucial coefficients (**A** and **B**) stem from one and the same matrix **Z**, divided for the base year, column-wise or row-wise, by total input or total output, respectively. This feature has two implications. First, for any single period of time direct relations between both coefficient sets follow from equations (2) and (10):

$$\mathbf{A} = \hat{\mathbf{x}}\mathbf{B}\hat{\mathbf{x}}^{-1} \quad \text{and} \quad \mathbf{B} = \hat{\mathbf{x}}^{-1}\mathbf{A}\hat{\mathbf{x}}. \tag{17}$$

Second, for any change over time, holding one set of coefficients stable, the following relations can be derived from equation (17):

$$\mathbf{A}_{t+1} = \hat{\mathbf{e}}\mathbf{A}_t\hat{\mathbf{e}}^{-1} \quad \text{for} \quad \mathbf{B}_{t+1} = \mathbf{B}_t \tag{18a}$$

and

$$\mathbf{B}_{t+1} = \hat{\mathbf{e}}^{-1}\mathbf{B}_t\hat{\mathbf{e}} \quad \text{for} \quad \mathbf{A}_{t+1} = \mathbf{A}_t \tag{18b}$$

where: $\hat{\mathbf{e}} = \hat{\mathbf{x}}_{t+1}\hat{\mathbf{x}}_t^{-1}$, i.e., the relative growth of total output (i.e., total input).

These relations ignited a rather confusing debate, mainly concentrating on terminology [5; 11; 6; 27; 13; 12]. Unfortunately, this debate did not contribute much to our understanding of the applicability of both models. In this respect, the most important conclusion from (18) is simply that one can expect to find a statistically significant difference in the stability of the two sets of coefficients, only when individual sectors show strongly uneven output growth, such as simulated in impact studies. Such conditions, however, are rarely encountered in practice. Sectoral rates of growth in real life only differ within certain bounds. Hence, the relative merits of the two models can only be deduced from their theoretical underpinnings and causal implications, especially in the case of impact studies.

As for causality, this runs opposite in both models. Loosely formulated: (final) demand drives the Leontief quantity model in which supply reacts with perfect (price) elasticity, whereas (primary) supply drives the Ghoshian quantity model with demand reacting with perfect (price) elasticity. In the latter case, this implies that cars may be driven without gas and factories may be run without essential inputs or even worse without labor. This was the prime reason why I labelled the supply-driven model as far more implausible than the demand-driven model [33].

In the ensuing discussion [21; 34], the theoretical underpinnings of both models were further clarified. The supply-driven model, which theoretically opposes the Leontief model most, explicitly assumes multiple outputs.[2] Hence, the economic derivation of both models is best started from

2. This is not recognized by Gruver [21], who bases his defence of the supply-driven model on the assumed base year coincidence of the iso-cost hyperplane and the iso-production hyperplane. However, in this interpretation of the supply-driven model, the slightest change in relative prices will lead to a corner solution which, as I argue [34], destroys Gruver's "small changes applications are okay" defense of the supply-driven model.

production functions with multiple inputs to suit the demand-driven model, as well as multiple outputs to suit the supply-driven model. For one technology, i.e., one single industry:

$$f(x_1, \ldots, x_I, y_1, \ldots, y_J) = 0 \tag{19}$$

where x_i represents the minimum quantity of input i needed given the outputs y_j and $x_{k,k \neq i}$, or, alternatively, where y_j represents the maximum quantity of output j that can be produced given the x_i and $y_{n,n \neq i}$. Subsequently, in the Leontief models equation (19) gets only one (homogeneous) output y, and in the Ghoshian models equation (19) gets only one (homogeneous) input x.

Under the assumption of perfect competition and profit-maximizing firms, purely opposite Leontief and Ghoshian quantity models may be derived as special cases from equation (19) [30]. The derivation starts with profit maximization, formalized as:

$$\max W = \sum_j p_j y_j - \sum_i p_i x_i. \tag{20}$$

Under perfect competition, in both (quantity) models, firms are confronted in (20) with given prices, both for outputs (p_j) and for inputs (p_i). In the demand-driven model, profit maximization becomes cost minimization as firms face a given price and demand for their single output, but operate in equation (19) with multiple inputs. In the supply-driven model, firms, with multiple outputs but a given price and supply of their single input, become revenue maximizers.

From a plausibility point of view, the economic implications of the "single homogeneous input" assumption and the "perfectly elastic demand for outputs" assumption of the supply-driven model are most ludicrous, especially in the case of impact studies. In the case of an aluminum shortage [5], the use of primary inputs in the shrinking aluminum-processing sectors (being exogenous) does not decrease, whereas it should along with production. And even more implausible, the production of aluminum-substitutes, such as other metals and plastics, does not increase as one would expect, but, in fact, even decreases along with the production of all other sectors. To handle supply-driven impact studies properly, an alternative approach, using so-called working-up or processing coefficients together with import and export substitution coefficients, is developed by Oosterhaven [33].

The supply-driven quantity model may, when causal interpretations are carefully avoided, only be helpful in descriptive analyses of relative forward linkage strength [4; 23] or in comparisons of economic structure over space and time [1; 14]. It may, however, definitely not be used to simulate the economic consequence of supply shocks or to produce supply-constrained economic projections.[3]

In principle, the causality and the assumptions involved in the price version of the Ghoshian model are as implausible as those of the quantity version. Demand-pull inflationary process-applications of the price model will produce results that are potentially as ridiculous as those of the Ghoshian quantity model.

In the "cost-push i-o price model" all sectors set their output prices independently and pass all cost increases on to their purchasers, who do not react. In the "demand-pull i-o price model"

3. The area of centrally planned economies was claimed as the prime candidate for applications of the supply-driven model in its original presentation by Ghosh [16]. Oosterhaven [33] shows that the same defects apply in this area, and he presents an alternative that combines fixed output (planning) coefficients with the Leontief production function in a linear programming context.

the opposite happens. All sectors set their *input* prices independently, passing any increase in revenue from rising output prices on to their suppliers, who do not react either. Hence, the cost-push price model may not be entirely plausible, but the demand-pull price model is much less plausible.

V. Concluding Evaluation

The contrast between the two price models, however, is not as large as that between the two quantity models, despite the fact that they are based on the same assumptions. The reason for this somewhat unexpected conclusion lies in the independence of the quantity and the price models (see Figure 1 and 2).

In applications of the quantity models the perfect substitution and perfect complementarity assumptions become, in fact, operational. So, in the supply-driven model cars may drive without gas and factories may run without labor. In applications of the demand-pull price model these consequences are latently present, but do not become operational, as quantities do not change. This is why the difference in plausibility between the two price models may be considered as smaller than that between the two quantity models.

To end with a positive note, it needs to be emphasized that all four models dealt with here, are extreme cases of multisectoral general equilibrium models. Hence, this paper may also be read as a plea to discard the economics of Figure 1 as well as that of Figure 2, and to replace it by the more familiar (negatively and positively sloped) demand and supply functions. Easily said, of course, but, empirically far more difficult to execute, especially in an interindustry setting.

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