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AN INTERREGIONAL LABOUR MARKET MODEL INCORPORATING VACANCY CHAINS AND SOCIAL SECURITY

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ABSTRACT This paper deals with three aspects of interregional demo-economic models which are important with respect to the analysis of regional labour market developments. First, attention is paid to forecasts of the exogenous regional variables, such as investments, exports, etc. Because data at the regional level are usually scarce, it is suggested to make these projections by means of a top-down model. Second, the social security sector is incorporated in the model framework, because changes in social security benefits, which are common in many Western countries nowadays, usually have substantial impacts on regional economic and demographic structures. Third, a vacancy chain model instead of a simple employment growth equation is tied to the standard demo-economic model framework. In this way not only employment growth but also migration, job mobility and related income effects can be taken into account.

1. INTRODUCTION

In the past decade or more regional labour market research activity has steadily increased. This increase is closely related to the explosion of unemployment in various countries during the seventies and eighties. A substantial part of labour market research is policy-oriented. It focusses on attempts to equalize regional unemployment and to attain intra-regional equilibrium in the sense of low and stable unemployment rates.

Two main fields of policy-oriented labour market research may be distinguished:

— Forecasting analysis of regional labour market developments.
— Impact analysis of policy-related phenomena.

Globally speaking, regional labour market developments are influenced by economic and social (particularly demographic) factors. Furthermore, economic policy usually has substantial impacts on both the economic and the demographic structures of regions. Therefore comprehensive analyses of labour market developments and of impacts of economic policy require models which consist of three basic components: the economic structure, the population structure and the labour market. The type of model which meets this requirement is the so-called demo-economic model, as can be seen in Figure 1 where the general structure of such a model is depicted.

Although the general structure of demo-economic models is well-described (see, among others Batey 1985), various methodological aspects have received relatively little attention. Therefore, some aspects are dealt with in this paper. The following issues are discussed.

In Section 2, attention will be paid to *forecasting* economic developments at the regional level. For the following reasons this will be done by linking regional projections to national projections, i.e. by means of a top-down model.

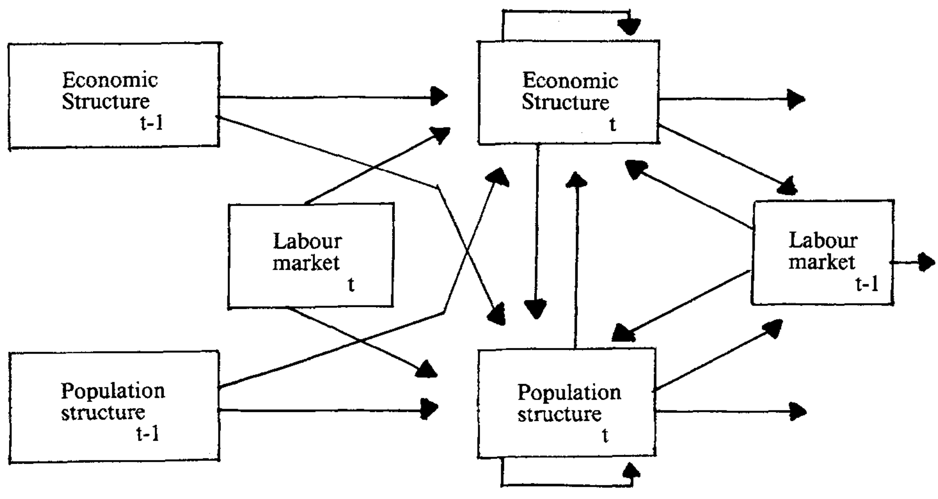

FIGURE 1. General Structure of a Demo-Economic Model

First, the data available at the regional level in most countries make it almost impossible to build adequate, up-to-date, genuine regional models (see also Klein and Glickman 1977). Secondly, in many countries yearly or even quarterly econometric forecasts are produced of various key variables, such as investments, consumption, employment, etc., at the national level. It is of great interest to analyze what the implications of these forecasts are at the regional level. A top-down approach is appropriate for this purpose.

Section 3 deals with a specific policy feature, viz. the *social security sector.* The reason for this interest is that at the regional level relatively little attention has been paid to this sector, whereas it may have substantial impacts on various aspects of the regional economy, in particular on its income formation and its labour market. Furthermore, a substantial reduction of the social security sector is presently under debate in various countries.

In Section 4 attention is paid to *vacancy chains.* In traditional demo-economic models it is assumed that new jobs are being filled by people that were unemployed in the same or in another region. In fact, however, a major part of all vacancies are filled by people who quit their jobs. Hence, chains of vacancies occur as a consequence of each job opening. Such chains may not be disregarded because workers filling vacancies often change both their employment sector and their region of employment (see Van Dijk and Oosterhaven 1985, for the case of the northern part of the Netherlands). Such changes lead to income changes and to migration.

Various aspects of the demo-economic framework will not be dealt with here because they have been extensively described elsewhere. In particular standard input-output analysis and demographic forecasting are left out of consideration. For the first we refer to, among others, Oosterhaven (1981) and for the second to, among others, Rogers (1975). However, in Section 5 a *complete* interregional demo-economic projection model, enlarged with the various aspects dealt with in this paper, will be presented.

2. DERIVATION OF REGIONAL ECONOMIC PROJECTIONS FROM NATIONAL FORECASTS

In this section attention is paid to making regional economic projections by linking regional projections to national forecasts.

Usually, the economic structure in demo-economic models is modelled by means of an input-output framework. Therefore, we assume here the availability of an interregional input-output model for at least two regions: the region under study and the remainder of the country. (The need for an interregional input-output model instead of a single regional framework will be described in Section 3.) Given an adequate input-output model, the implications of national projections for the regional economy can be derived in a straightforward way when the implications of the national projections for the various categories of final demand for the regions distinguished are known. In the case of a model with the region under study and the remainder of the country as the only regional units, only the consequences for one unit (usually the region under study) must explicitly be derived because the projection for the other unit can be obtained as the difference of the national and the derived regional projection. Anyhow, the projection problem under consideration here is basically a problem of estimating the relationships between the categories of final demand in an interregional input-output model and the projections of the corresponding variables at the national level.

As mentioned in the introduction, the derivation of regional forecasts will be obtained by means of a top-down model. In this kind of approach regional variables are related to national variables without a feed-back of the former on the latter. A top-down relationship can formally be represented as:

$$f(y^r_t, y^r_{t-1}, \ldots, y^r_{t-q}, x^r_t, x^r_{t-1}, \ldots, x^r_{t-p}, x^n_t, x^n_{t-1}, \ldots, x^n_{t-s}, \theta) = \varepsilon_t \qquad (1)$$

where f = some real-valued function,
y^r = a vector of regional endogenous variables,
x^r = a vector of regional exogenous variables,
x^n = a vector of national exogenous variables,
θ = a vector of parameters,
ε = a vector of random disturbances.

In concrete cases the actual form of (1) is dependent on theoretical considerations and the data available. Some possible forms are enumerated below.

The most simple form, which is used when information on only y^r_t and the corresponding national variable y^n_t is available for some period in the recent past, reads as:

$$\hat{y}^r_{\tilde{t}} = \alpha\, \hat{y}^n_s$$

where $\tilde{t}$ denotes some future period, $\hat{y}^n_s$ is the exogenous forecast of the national variable and α is obtained as:

$$\alpha = \frac{y^r_{t-p}}{y^n_{t-q}}$$

where both y^r_{t-p} and y^n_{t-q} are observed realizations. It should be noted that usually $p = q$ and $\tilde{t} = s$. However, for reasons of the availability of data, s and q may be different from $\tilde{t}$ and p, respectively. When information on y^r_t is missing but data on a highly correlated variable, say $\tilde{y}^r_t$, are available, α may be derived as:

$$\alpha = \frac{\bar{y}^r_{t-p}}{\bar{y}^n_{t-q}}$$

When a time series on both y^r_t and y^n_t is available the variable $z_t = y^r_t/y^n_t$ can be modelled by means of an autoregressive integrated moving average (ARIMA) model. (For various model forms, estimation, diagnostic checking, model modification and forecasting, see Box and Jenkins 1976; for applications in spatial research, see Folmer 1985.) From the ARIMA model, forecasts $\hat{z}_{\tilde{t}}$ of $z_{\tilde{t}}$ for future $\tilde{t}$'s can be obtained. A forecast of $y^r_{\tilde{t}}$ can then be obtained as:

$$\hat{y}^r_{\tilde{t}} = \hat{z}t\ \hat{y}^n_{\tilde{t}}$$

where $\hat{z}_{\tilde{t}}$ is the forecast of $z_{\tilde{t}}$ for the future $\tilde{t}$.

When information is available on all the relevant variables, model (1) may be specified as a multivariate time series model.

When limited information is available on the exogenous national variable and on a related explanatory variable it may be possible to combine both types of information. As an example, consider the variable "number of civil servants" (cs) and assume that cs^r and cs^n are both known for some period q in the past. Furthermore, suppose that the national and the regional populations (P) are also known at q and t. If cs^r_t is (additively) determined by autonomous developments, which are equally strong at the national and the regional level, and by the relative population growth in region r, $cs^r_{\tilde{t}}$ can be obtained as:

$$\hat{cs}^r_{\tilde{t}} = \hat{cs}^n_{\tilde{t}} \left[\frac{cs^r_t}{cs^n_t} + \frac{\Delta P^r_{q\text{-}\tilde{t}}}{\Delta P^n_{q\text{-}\tilde{t}}} \right]$$

where ΔP denotes the population increase.

The top-down approach can be used to obtain projections of the various final demand categories investments, foreign exports, inventories, government consumption and household consumption by non-wage earners, in particular those who benefit from social security. When projections for the final demand categories in the interregional input-output model have been obtained, projections of the endogenous variables, in particular employment, can be obtained in the usual way (cf. Oosterhaven 1981).

3. INCORPORATING SOCIAL SECURITY INTO A DEMO-ECONOMIC FRAMEWORK

Introduction

In most Western countries public spending on social insurance and welfare programmes (to be denoted simultaneously as social security programmes) has increased rapidly during the past twenty years. In the Netherlands, for example, expenditures on these programs as a percentage of the Net National Income amounted to 7.3% in 1950, 11.2% in 1960, 19.6% in 1970, 29.3% in 1980 and 33.4% in 1984. Four main reasons for the increase can be distinguished:

- — Demographic factors, in particular the ageing of the population.
- — The introduction of new social security programmes.
- — The increase of benefit levels.
- — The growth of unemployment.

The social security programmes have two basic objectives (cf. Danziger and Haveman 1981). First, they replace income losses from events that are outside an individual's control and, second, they assure a minimum level of economic support to those who have insufficient income.

In addition to these intended effects for individuals, social security programmes may have various more or less unintended effects, both at the micro, the regional and the macro levels of the economy. The unintended micro effects on such aspects as the supply of labour, private savings and the distribution of income have quite well been investigated (cf. Danziger and Haveman 1981 for an overview). Investigations of the macro economic effects have primarily focused on the role of social security programmes as automatic stabilizers (see, among others, Eilbott 1966; Von Furstenberg 1976).

The interest in social security at the micro and macro levels is in contrast with the attention paid to it at the regional level. Among the small number of papers dealing with this issue are the ones by Oaxaca and Taylor (1983), Batey and Madden (1983), Madden and Batey (1983).

This scanty interest is surprising because social security programmes may have impacts on various aspects of the local economy. The aspects which are most likely to be affected are real disposable income, production, employment, the labour force, the unemployment rate and the local population. Furthermore, the impacts may differ across regions for the following reasons (see also Oaxaca and Taylor 1983):

— Differences in social security programmes, in particular in benefit levels. In some countries, like the Netherlands, differences in benefit levels hardly exist, but in, for instance, the U.S. these differences may be substantial. The differences in impacts over regions with different benefit levels may be enhanced by nonlinearities.
— The redistributive nature of social security programmes influences the interregional formation of incomes.
— Differences in the structures of the regional economies. For example, regions with a relatively large export sector are likely to experience smaller effects than regions with a relatively large sector producing consumption goods for regional or national markets with high individual income elasticities of demand.
— Local income multipliers are not identical across regions because of differences in the levels of self-sufficiency of regional markets.
— Differences in migration flows as reactions to the first three alternations of the relative economic attractiveness of different regions. The migration flows in their turn may mitigate or enhance the initial impacts of the social security programmes. In this way circular chains of effects may arise.

This brief discussion of the impacts of social security programmes on regional economies shows that it is of great importance to incorporate the social security sector in regional projection models. This holds in particular in situations where these programmes are under discussion, as in the Netherlands nowadays. Therefore, we will indicate in which way the social security sector may be incorporated into a demo-economic model.

In the framework of a demo-economic model, two kinds of effects of changes in social security programmes need to be distinguished: viz. *consumption*-induced effects and *migration*-induced effects. The former can be subdivided into direct regional impacts, which are functions of the regional multipliers, and indirect effects, which arise in regions other than the one under consideration and are transmitted through the national market.

The indirect impacts are a function of:

— The regional multiplier.

— The importance of the region as a producer for the national market.
— The extent to which commodities are purchased at a national market.
— The sensitivity of nationally produced commodities to national income.

Both the direct and indirect consumption effects have consequences for regional unemployment which in turn affect migration flows to and from the region under study. Migration in its turn affects regional consumption and employment (cf. Van Dijk and Oosterhaven 1985) which leads to second-order migration effects, and so on.

From these observations, the following conclusions may be drawn concerning the structure of a demo-economic model, intended to monitor impacts of changes in social security programmes:

— First, the demo-economic model should be *interregional* of nature in the sense that it takes into account the relationship between the regional economy and the economy of remainder of the country. As mentioned above, various forms of spatial aggregation can be distinguished in this connection. In the sequel an R-dimensional model is assumed.
— Second, the social security sector and its relationship to migration and employment should be explicitly incorporated in the model framework. The social security benefits can be split up into several categories, such as unemployment benefits, pensions, etc. For the sake of simplicity only two categories will be distinguished in this paper: unemployment benefits and social security benefits for non-actives, such as retired people.

The extended demo-economic model

In this section the demo-economic model with I sectors described by Batey (1985) will be extended on the basis of the conclusions of the introduction to Section 3. Before going into detail it is of importance to remark that the following types of endogenous income changes are usually distinguished in intersectoral, input-output based, demo-economic models:

— Increases in labour incomes accruing to resident workers, i.e. *intensive* income growth (cf. Miernyk et al. 1967; Tiebout 1969).
— New labour incomes accruing to migrants or local unemployed, i.e. *extensive* income growth (cf. Miernyk et al. 1967; Tiebout 1969).
— Changes in social security benefits when unemployed become employed.

The combined change in social security benefits and its counterpart of extensive income growth is denoted as *redistributive* income change (cf. Blackwell 1978).

It should be noted that the income change as a consequence of transition from one social security sector to another is also of importance in this paper. This type of income change will be denoted as *transitional.*

These types of income changes are distinguished because of the different consumption expenditure coefficients that have to be used in each case. To intensive income growth *marginal* coefficients have to be applied; to the other categories group-specific *average* coefficients.

Oosterhaven (1981) presents an interregional input-output model that incorporates intensive, extensive and redistributive income changes. A slightly different regional version of this model is presented in Batey (1985) as the most comprehensive of a family of ten demo-economic models. These approaches will be extended here to incorporate the social security sector.

It should be noted that in order to get a more accurate description of the dynamics of the economic processes the interrelated increases in wage rates and in labour productivity are explicitly introduced. In this way it is possible to model intensive income growth explicitly, instead of defining it as a residual factor, as in Batey (1985).

The model contains I sectors and R regions. Therefore, it has to be presented in matrix notation. Hence, unless stated otherwise, all symbols from now on refer to vectors (in small type), diagonal matrices (in small type written as $\hat{.}$) or matrices (in capitals). It should be noted that when the dimension of a vector is IR or the order of a matrix is $IR \times IR$ the variables, respectively coefficients, are defined per region per sector; when the dimension is R or the order is $R \times R$ the definitions are with respect to regions and when the dimension is I or the order $I \times I$ the definitions refer to sectors.

The model reads as follows:

$$\begin{aligned} \Delta \boldsymbol{x} = \boldsymbol{A}\, \Delta \boldsymbol{x} + \boldsymbol{H}^m\, \Delta \boldsymbol{y}^{int} + \boldsymbol{H}^a\, \Delta \mathrm{y}^{ext} \\ + \boldsymbol{H}^u\, \hat{\boldsymbol{s}}^u\, \Delta \boldsymbol{u} + \boldsymbol{H}^n\, \hat{\boldsymbol{s}}^n\, \Delta \boldsymbol{n} + \Delta \boldsymbol{f}^{ex} \end{aligned} \tag{2}$$

$$\Delta \boldsymbol{y}^{int} = \boldsymbol{C}\, \Delta \hat{\boldsymbol{w}}\, (\boldsymbol{e}_{-1} + \Delta \boldsymbol{e}) \tag{3}$$

$$\Delta y^{ext} = C\, \hat{w}_{-1}\, \Delta e \tag{4}$$

$$\Delta u = -M^u\, \Delta e + \Delta u^{ex} \tag{5}$$

$$\Delta e = \hat{\boldsymbol{k}}\, \Delta x + \Delta \hat{\boldsymbol{k}}\, x_{-1} \tag{6}$$

where: $\Delta \boldsymbol{x}$ = IR-vector with changes in gross production;

$\boldsymbol{x}_{-1}$ = IR-vector with lagged gross production levels;

$\Delta \boldsymbol{y}$ = R-vector of income growth where *int* denotes intensive and *ext* extensive;

$\Delta \boldsymbol{u}$ = R-vector with the total change in the number of unemployed;

$\Delta \boldsymbol{n}$ = R-vector with the change in the exogenous number of people who receive social security payments other than unemployment benefits;

$\Delta \boldsymbol{e}$ = IR-vector with changes in employment;

$\boldsymbol{e}_{-1}$ = IR-vector with lagged employment levels;

$\Delta \boldsymbol{f}^{ex}$ = IR-vector with changes in exogenous final demand;

$\boldsymbol{A}$ = $IR \times IR$ matrix with intermediate input-output coefficients;

$\boldsymbol{C}$ = $R \times IR$-commuting matrix where the (r, r)th I-dimensional row vector denotes the proportions of workers living and working in region r in the various sectors and the (r, s)th off-diagonal vector the proportions of workers living in region r and working in region s in the various sectors (It should be noted that $\boldsymbol{i}'\, \boldsymbol{C} = \boldsymbol{i}'$, i.e. $\boldsymbol{C}$ has column sums equal to one);

$\Delta \hat{\boldsymbol{w}}$ = IR-diagonal matrix with the increases in average wage rates;

$\hat{\boldsymbol{w}}_{-1}$ = IR-diagonal matrix with lagged average wage rates;

$\boldsymbol{H}$ = $IR \times IR$ matrix with consumption expenditures coefficients, where m denotes marginal with respect to $\Delta \boldsymbol{y}^{int}$ and a, u and n denote average with respect to $\Delta \boldsymbol{y}^{ext}$, $\Delta \boldsymbol{u}$ and $\Delta \boldsymbol{n}^{ex}$;

$\boldsymbol{M}^u$ = $R \times IR$-matrix, where the R I-diagonal rows denote the probabilities that new jobs in sector i and region r are taken up by unemployed from the same region, and where the off-diagonal rows denote the probabilities that these jobs are taken up by unemployed from other regions (It should be noted that $\boldsymbol{i}'\, \boldsymbol{M}^u = \boldsymbol{i}'$, i.e. $\boldsymbol{M}^u$ has column sums equal to one);

$\hat{\boldsymbol{k}}$ = IR-diagonal matrix with employment/production coefficients;

$\Delta\hat{k}$ = IR-diagonal matrix with the *decreases* in employment/production coefficients, i.e. with the reciprocals of the increases in labour productivity;

$\hat{s}$ = R-diagonal matrix with average social security payments per region of residence with u referring to unemployment benefits and n referring to benefits from the remaining social security sector;

i = vector of ones, i.e. a summation vector of the appropriate dimension;

$'$ = transpose.

The following remarks are important here. First, substitution of (4) and (5) into (2) shows that with regard to redistributive income growth marginal consumption expenditures are applied, viz. $(H^a\ C\ \hat{w}_{-1} - H^u\ \hat{s}^u\ M^u)\ \Delta e$. (The need to apply marginal coefficients to redistributive income growth is described in Blackwell 1978). Second, the sum of the intensive and extensive income growth by region of workplace must equal the total labour income growth. This condition is met here, which can be seen as follows:

$$\Delta\hat{w}\ (e_{-1} + \Delta e) + \hat{w}_{-1}\ \Delta e = \Delta\hat{w}\ e_{-1} + \hat{w}\ \Delta\ e =$$

$$\hat{w}\ \hat{k}\ \Delta x + (\Delta\hat{w}\ \hat{k}_{-1} + \hat{w}\ \Delta\hat{k})\ x_{-1} = \hat{h}^w\ \Delta x + \Delta\hat{h}^w\ x_{-1}$$

where: $\hat{h}^w$ = IR-diagonal matrix with labour income/production coefficients which are by definition equal to $\hat{w}\ \hat{k}$.

When the labour income/production coefficients are stable over time, the employment decreasing and wage increasing effects of rises of labour productivity on total labour income balance, i.e. $\Delta\hat{h}^w$ is the zero matrix. This implies that the *total* increase is solely due to the rise in gross production ($\hat{h}^w\ \Delta x$), as in Batey (1985).

Model (2)-(6) can be refined by specifying Δu and Δn as endogenous variables. This will be done in Section 5. Furthermore, in (5) only net employment growth is modelled. In order to make adequate labour market projections, other causes of vacancies than net employment growth should also be taken into account. Furthermore, effects of the filling of vacancies on consumption expenditures and the demographic structure should be considered. As mentioned above, the origin of vacancies will be described in Section 4.

4. VACANCY CHAINS AND THEIR DEMO-ECONOMIC SIGNIFICANCE

The intra-organizational significance of vacancy chains is extensively dealt with by White (1970). He also indicates how the taking up of vacancies may be modelled as a Markov-process. MacKinnon (1975) and MacKinnon and Rogerson (1980) describe how vacancy chains may be incorporated in migration models as one of the main explanatory factors. We will extend the work of the two last-mentioned authors and show how a vacancy model can be tied into a demo-economic framework.

We assume that vacancies in sector i in region s *occur* for the following reasons:

— Workers with a job in this sector may fill a vacancy in sector j in region s. This transition process from one sector to another may be described by a matrix T^w with transition probabilities, which depend on the qualifications of the workers in this sector, their mobility and the qualifications required in the sector of destination.

— New jobs may be created in the sector at hand. The number equals $\hat{k}\Delta x$.

However, these new jobs are partially filled by workers who are dismissed from their present position *without* becoming unemployed. In the framework of the present model dismissals are assumed to be a function of a change in labour productivity. For the ease of understanding, the employment decreasing effect of labour productivity increases ($\Delta\hat{k}\ x_{-1}$, cf. Equation (6)) will be written as $-\ \Delta\hat{z}\ e_{-1}$, where $\Delta\hat{z}$ is an *IR*-diagonal matrix with the increases in production per worker. We assume that per regional sector a fraction ($\hat{g}$) of the redundant workers is not dismissed but is taking up the newly created jobs.

— Finally, vacancies come into being because workers may retire (early) or die. This process is described by retirement and death rates. These rates *inter alia* depend on the age structure of the labour force concerned.

Hence, the following equation defines the total number of vacancies to be filled in any period under consideration:

$$v = T^w\ v + \hat{k}\ \Delta x - \hat{g}\ \Delta\hat{z}\ e_{-1} + \hat{r}\ e_{-1} + \hat{d}^w\ e_{-1} \tag{7}$$

where: v = *IR*-vector of vacancies to be filled;
T^w = *IR* × *IR*-matrix with transition probabilities for workers between sectors and regions;
$\hat{g}$ = *IR*-diagonal matrix with the internal replacement ratios of those workers which are redundant because of labour productivity increases;
$\Delta\hat{z}$ = *IR*-diagonal matrix with increases in the production per worker;
$\hat{r}$ = *IR*-diagonal matrix with (early) retirement rates;
$\hat{d}^w$ = *IR*-diagonal matrix with death rates for workers.

Vacancies may be *taken up* by workers from the same or from other sectors or by unemployed. For the sake of simplicity it is assumed that economically inactives do not fill vacancies. The filling up of vacancies by unemployed can be described by transition probabilities for unemployed living in region *r* who find a job in sector *j* in region *s*. These transition probabilities again depend on the job requirements in sector *j* in region *s* and on the qualifications of the unemployed in region *r*, as well as on the mobility of the unemployed. Hence, unemployment per region will be affected by vacancies in the following way:

$$\Delta\ \ddot{u} = -T^u\ v \tag{8}$$

where: $\Delta\ddot{u}$ = *R*-vector with the changes in the number of unemployed due to the occurrence of vacancies;
T^u = *R* × *IR*-matrix, where the *i*-th row of the *R* *I*-diagonal rows contains the probability that sector *i* fills its vacancies with local unemployed and the (*r, s*)th off-diagonal row contains the probabilities that vacancies in the various sectors in region *s* are filled by unemployed from region *r*.

The solution of (7)-(8) is straightforward:

$$v = (I - T^w)^{-1}\ (\hat{k}\ \Delta x - \hat{g}\ \Delta\hat{z}\ e_{-1} + \hat{r}\ e_{-1} + \hat{d}^w\ e_{-1}) \tag{9}$$

$$\Delta\ddot{u} = -T^u\ (I - T^w)^{-1}\ [\hat{k}\ \Delta x + (\hat{r} + \hat{d}^w - \hat{g}\ \Delta\hat{z})\ e_{-1}] \tag{10}$$

The part of the vacancies that is filled up by workers equals $T^w\ v$ and the part filled up by unemployed equals $T^u\ v$.

It should be noted that there exists a permanent pool of vacancies because of labour market frictions. For simplicity's sake, we will assume that the numbers of vacancies per regional sector at the beginning and at the end of the period under consideration are equal. Hence, we assume that all additional vacancies are being filled up. This implies that:

$$\boldsymbol{i}' \boldsymbol{T}^w + \boldsymbol{i}' \boldsymbol{T}^u = \boldsymbol{i}' \tag{11}$$

Condition (11) has an important consequence for the values of the multipliers in (10), viz. that the column sums of $\boldsymbol{T}^u (\boldsymbol{I} - \boldsymbol{T}^w)^{-1}$ are equal to one. This implies:

$$\boldsymbol{i}' \Delta\tilde{\boldsymbol{u}} = -\boldsymbol{i}' [\hat{\boldsymbol{k}} \Delta\boldsymbol{x} + (\hat{\boldsymbol{r}} + \hat{\boldsymbol{d}}^w - \hat{\boldsymbol{g}} \Delta\hat{\boldsymbol{z}}) \boldsymbol{e}_{-1}] \tag{12}$$

This means that the *total* number of unemployed filling up vacancies precisely equals the *total* number of exogenously created vacancies. It should be noted, however, that this does not imply that the unemployed themselves fill these exogenous vacancies, as was the case in Equation (5).

This vacancy-chain model has been applied to the situation in the northern part of the Netherlands (cf. Van Dijk and Oosterhaven 1985). In the next section this model will be integrated with model (2)-(6).

5. THE COMPLETE INTERREGIONAL DEMO-ECONOMIC MODEL

In this Section we will integrate the partial models of the foregoing sections. We will first pay attention to production changes ($\Delta\boldsymbol{x}$). This variable equals the sum of:

(1) Changes of exogenous final demand ($\Delta\boldsymbol{f}^{ex}$), which my be forecasted by means of the methods described in Section 2.
(2) Changes of intermediate demand ($\boldsymbol{A} \Delta\boldsymbol{x}$).
(3) Changes of endogenous consumption demand originating from:
   - Increases of labour incomes per job ($\boldsymbol{H}^m \Delta\boldsymbol{y}^{int}$);
   - Changes in labour incomes because of job changes or the transition from unemployed to employed and vice versa ($\boldsymbol{H}^a \Delta\boldsymbol{y}^{ext}$);
   - Changes of unemployment benefits because of people entering or leaving the regional pools of unemployed ($\boldsymbol{H}^u \hat{\boldsymbol{s}}^u \Delta\boldsymbol{u}$).
   - Changes of social security payments other than unemployment benefits. These changes arise as a consequence of the growth or decline of the number of sick or retired people, etc. ($\boldsymbol{H}^n \hat{\boldsymbol{s}}^n \Delta\boldsymbol{n}$).

These changes can formally be summarized as follows:

$$\Delta\boldsymbol{x} = \boldsymbol{A} \Delta\boldsymbol{x} + \boldsymbol{H}^m \Delta\boldsymbol{y}^{int} + \boldsymbol{H}^a \Delta\boldsymbol{y}^{ext} + \boldsymbol{H}^u \hat{\boldsymbol{s}}^u \Delta\boldsymbol{u} + \boldsymbol{H}^n \hat{\boldsymbol{s}}^n \Delta\boldsymbol{n} + \Delta\boldsymbol{f}^{ex} \tag{13}$$

Next, we turn to the net change in the number of jobs per regional sector ($\Delta\boldsymbol{e}$). This variable depends positively on production growth ($\hat{\boldsymbol{k}} \Delta\boldsymbol{x}$) and negatively on labour productivity growth ($\Delta\hat{\boldsymbol{z}} \boldsymbol{e}_{-1}$):

$$\Delta\boldsymbol{e} = \hat{\boldsymbol{k}} \Delta\boldsymbol{x} - \Delta\hat{\boldsymbol{z}} \boldsymbol{e}_{-1} \tag{14}$$

Vacancies per regional sector ($\boldsymbol{v}$) come into being because of leaves to take up other vacancies ($\boldsymbol{T}^w \boldsymbol{v}$), new job openings ($\hat{\boldsymbol{k}} \Delta\boldsymbol{x} - \hat{\boldsymbol{g}} \Delta\hat{\boldsymbol{z}} \boldsymbol{e}_{-1}$), retirements into economic inactivity ($\hat{\boldsymbol{r}} \boldsymbol{e}_{-1}$) and decreases ($\hat{\boldsymbol{d}}^w$ e_{-1}):

$$\boldsymbol{v} = \boldsymbol{T}^w \boldsymbol{v} + \hat{\boldsymbol{k}} \Delta\boldsymbol{x} + (\hat{\boldsymbol{r}} + \hat{\boldsymbol{d}}^w - \hat{\boldsymbol{g}} \Delta\hat{\boldsymbol{z}}) \boldsymbol{e}_{-1} \tag{15}$$

The number of unemployed per region of living may change ($\Delta\boldsymbol{u}$) for a variety of reasons:

— Schoolleavers and other potential workers may enter the labour market and become entitled to unemployment benefits, possibly after migration.
— Redundant workers may be dismissed *without* being internally replaced. Furthermore they may migrate (see also Equation (7)).
— Unemployed may fill a vacancy, possible after migration: $\boldsymbol{T}^u \boldsymbol{v}$.
— Unemployed may leave the labour market and become inactive, e.g. because they stop searching for a job or because they reach the retirement age: $\hat{\boldsymbol{q}}\, \boldsymbol{u}_{-1}$.
— Unemployed may die or lose their entitlement to any social security payment: $\hat{\boldsymbol{d}}^u\, \mathrm{u}_{-1}$.

Formally:

$$\Delta \boldsymbol{u} = (\boldsymbol{M}^b)'\, \hat{\boldsymbol{b}}^u\, \mathrm{p}_{-1} + (\boldsymbol{M}^g)'\, (\boldsymbol{I} - \hat{\boldsymbol{g}})\, \Delta\hat{\boldsymbol{z}}\, \boldsymbol{e}_{-1} - \boldsymbol{T}^u\, \boldsymbol{v} - \hat{\boldsymbol{q}}\, \boldsymbol{u}_{-1} - \hat{\boldsymbol{d}}^u\, \mathrm{u}_{-1} \quad (16)$$

where: $\hat{\boldsymbol{b}}^u\, \mathrm{p}_{-1}$ = R-vector with entrants to the labour market, which is a function of lagged population in the region of origin;
$\boldsymbol{M}^b$ = RXR-matrix, with m^{rr} denoting the probability that an entrant to the labour market stays in the region of origin, and m^{rs} denoting the probability that the entrant migrates from region r to region s. $\boldsymbol{M}^b\, \boldsymbol{i} = \boldsymbol{i}$, i.e. the row sums are equal to one;
$\Delta\hat{\boldsymbol{z}}\, \boldsymbol{e}_{-1}$ = IR-vector with the numbers of redundant workers due to labour productivity increases;
$(\boldsymbol{I} - \hat{\boldsymbol{g}})$ = IR-diagonal matrix with the fractions of redundant workers that are not internally replaced;
$\boldsymbol{M}^g$ = $IR \times R$-matrix where the R I-diagonal columns contain the probabilities that the dismissed workers stay in the region of origin and the (r, s)th off-diagonal column contains the probabilities that the dismissed workers in the various sectors migrate from region r to region s.

The number of people per region of residence, who receive social security payments other than unemployment benefits, may change ($\Delta \boldsymbol{n}$) for partly similar reasons as in the previous case:

— New categories may become entitled to such payments, possibly after migration: $(\boldsymbol{M}^n)'\, \hat{\boldsymbol{b}}^n\, \mathrm{p}_{-1}$.
— Workers may retire, possibly after migration: $(\boldsymbol{M}^r)'\, \hat{\boldsymbol{r}}\, \boldsymbol{e}_{-1}$. (See the corresponding term in Equation (15)).
— Unemployed may retire, possibly after migration: $(\boldsymbol{M}^q)'\, \hat{\boldsymbol{q}}\, \boldsymbol{u}_{-1}$. (See the corresponding term in Equation (16)).
— Entitled persons may die or lose their entitlements: $\hat{\boldsymbol{d}}^n\, \mathrm{n}_{-1}$.

Formally:

$$\Delta \boldsymbol{n} = (\boldsymbol{M}^n)'\, \hat{\boldsymbol{b}}^n\, \mathrm{p}_{-1} + (\boldsymbol{M}^r)'\, \hat{\boldsymbol{r}}\, \boldsymbol{e}_{-1} + (\boldsymbol{M}^q)'\, \hat{\boldsymbol{q}}\, \boldsymbol{u}_{-1} - \hat{\boldsymbol{d}}^n\, \mathrm{n}_{-1} \quad (17)$$

where $\boldsymbol{M}^n$, $\boldsymbol{M}^r$ and $\boldsymbol{M}^q$ are defined analogously to $\boldsymbol{M}^b$.

Finally, we need to specify the two types of labour income changes. We assume that for all categories of workers (i.e. newly employed, employed migrants and those who continue working in the same regional sector) the same wage rates and commuting patterns apply. (It should be noted that this assumption is rather restrictive. For instance, wages of native, newly employed are likely to differ from the wages of employed migrants.) As a consequence of the assumption the extensive labour income effects of people entering or quitting jobs are all measured in the same wage rate of the regional sector at hand.

The number of people entering new jobs equals the number of vacancies (v) and the number of people quitting jobs, equals $T^w v + (I - \hat{g}) \Delta \hat{z} e_{-1} + (\hat{r} + \hat{d}^w) e_{-1}$. Therefore extensive labour income growth can be specified as:

$$\Delta y^{ext} = C \hat{w}_{-1} [v - T_v^w - (\Delta \hat{z} - \hat{g} \Delta \hat{z} - \hat{r} - \hat{d}^w) e_{-1}] \tag{18}$$

Furthermore, intensive labour income growth need not be redefined. So:

$$\Delta y^{int} = C \Delta \hat{w} (e_{-1} + \Delta e) \tag{19}$$

Model (13)-(19) may be summarized by putting all current endogenous variables at the left hand side of the equation signs and the lagged endogenous and the exogenous variables on the right hand side. Before doing so the simple employment equation (14) is substituted into the intensive income growth equation (19). The resulting model structure is given in the matrix equation depicted in Figure 2. The upper left block of this matrix contains the description of the economic system and the lower right contains the demographic system. The off-diagonal blocks contain the economic-demographic interaction relations. The right-hand column contains exogenous and lagged demographic and economic variables.

The entire system may of course be solved by inverting the large partitioned matrix given in Figure 2. However, when we take advantage of the large number of zero submatrices, the solution may also be obtained in a simple step-wise way. First (15) is solved, viz.

$$v = (I - T^w)^{-1} \{\hat{k} \Delta x + (\hat{r} + \hat{d}^w - \hat{g} \Delta \hat{z}) e_{-1}\} \tag{20}$$

Next, (20) is substituted into (18) and (16). This gives:

$$\Delta y^{ext} = C \hat{w}_{-1} (\hat{k} \Delta \hat{x} - \Delta \hat{z} e_{-1}) \tag{21}$$

$$\begin{aligned} \Delta u = {} & (M^b)' \hat{b}^u p_{-1} + (M^g)' \Delta \hat{z} e_{-1} - (\hat{g} + \hat{d}^u) u_{-1} \\ & - T^u (I - T^w)^{-1} \{\hat{k} \Delta x + (\hat{r} + \hat{d}^w - \hat{g} \Delta \hat{z}) e_{-1}\} \end{aligned} \tag{22}$$

Note that the assumption of equal wage rates and commuting patterns for all people taking up new vacancies leads to the simple form of (21).

Next, (17), (19), (21) and (22) are substituted into (13), which is then rearranged and inverted. This gives:

$$\begin{aligned} \Delta x = {} & [I - A - H^m C \Delta \hat{w} \hat{k} - H^a C \hat{w}_{-1} \hat{k} + H^u \hat{s}^u T^u (I - T^w)^{-1} \hat{k}]^{-1} \\ & [\Delta f^{ex} + H^m C \Delta \hat{w} (e_{-1} - \Delta \hat{z} e_{-1}) - H^a C \hat{w}_{-1} \Delta \hat{z} e_{-1} \\ & + H^u \hat{s}^u \{(M^b)' \hat{b}^u p_{-1} + (M^g)' (I - \hat{g}) \Delta \hat{z} e_{-1} - (\hat{q} + \hat{d}^u) u_{-1} \\ & - T^u (I - T^w)^{-1} (\hat{r} + \hat{d}^w - \hat{g} \Delta \hat{z}) e_{-1}\} + H^n \hat{s}^n \{(M^n)' \hat{b}^n p_{-1} \\ & + (M^r)' \hat{r} e_{-1} + (M^q)' \hat{q} u_{-1} - \hat{d} n_{-1}\}] \end{aligned} \tag{23}$$

The inverse between the first pair of square brackets indicates the feedback mechanism of the model. The second term between square brackets displays all exogenous and lagged endogenous variables. Except for the first variable in the second term (Δf^{ex}), all other variables refer to exogenous or lagged endogenous changes in consumption demand. They refer successively to consumption demand changes caused by:

- Labour income increases that accrue to the workers who, given the prevailing labour productivity, are needed to produce the old level of output.

Equation number	Dimensions of matrices and vectors: IR	IR	R	R	R	R	Endogenous variables		Exogenous variables and lagged endogenous variables
13	$(I - A)$	$\mathbf{0}$	$-H^m$	$-H^a$	$-H^u\hat{s}^u$	$-H^n\hat{s}^n$	Δx		Δf^{ex}
15	$-\hat{k}$	$(I - T^w)$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	v		$(\hat{r} + \hat{d}^w - \hat{g}\,\Delta\hat{z})\, e_{-1}$
18	$\mathbf{0}$	$Cw_{-1}(I - T^w)$	I	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	Δy^{ext}	$=$	$-C\,\hat{w}_{-1}\,\{(I - \hat{g})\,\Delta\hat{z} + \hat{r} + \hat{d}^w\}\, e_{-1}$
19	$-C\,\Delta\hat{w}\,\hat{k}$	$\mathbf{0}$	$\mathbf{0}$	I	$\mathbf{0}$	$\mathbf{0}$	Δy^{int}		$C\,\Delta\hat{w}\,(e_{-1} - \Delta\hat{z}\, e_{-1})$
16	$\mathbf{0}$	T^u	$\mathbf{0}$	$\mathbf{0}$	I	$\mathbf{0}$	Δu		$(M^b)'\,\hat{b}^u\, p_{-1} + (M^g)'\,(I - \hat{g})\,\Delta\hat{z}\, e_{-1} - (\hat{q} + \hat{d}^u)\, u_{-1}$
17	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	I	Δn		$(M^n)'\,\hat{b}^n\, p_{-1} + (M^r)'\,\hat{r}\, e_{-1} + (M^q)'\,\hat{q}\, u_{-1} - \hat{d}^n\, n_{-1}$

FIGURE 2. The Overall Structure of the Model

— Labour income decreases that are due to the decreases in employment caused by the increase in production per worker.
— Changes in unemployment benefits caused by, respectively, entrants at the labour market; unemployment growth as a consequence of the dismissal of redundant workers, who can *not* be internally replaced; unemployment decline because of unemployed who take up vacancies as a consequence of retirements and deaths; prevention of the decline of unemployment because of redundant workers being replaced internally.
— Changes in inactive payments caused by, respectively, new entrants; workers and unemployed who retire; and deaths.

A change in one of the above-mentioned components sets off four feedback loops (see the inverted term of Equation (23)), which are related to respectively:
— Intermediate demand changes;
— Consumption demand from wage increases on newly created jobs;
— Consumption demand from wages on newly created jobs (measured in lagged wage levels);
— A negative feedback on consumption demand because unemployed lose their unemployment benefits when they get a job.

6. CONCLUSION

In many Western countries the most important goal of regional policy has been the reduction of differences in unemployment rates (cf. Oosterhaven and Folmer 1983, for the Netherlands). The design and evaluation of this kind of policy *inter alia* requires models, which adequately describe the working of the regional labour market. As the state of a regional labour market is primarily determined by economic and demographic variables and their interactions, a demo-economic model provides an adequate framework to monitor labour market developments.

In this paper it has been shown that a demo-economic model to analyse labour market developments should be interregional. Furthermore, as data at the regional level usually are scarce, it has been suggested to make forecasts of the various exogenous variables by means of a top-down model. It has also been argued that the social security sector should be explicitly incorporated in the model. For the sake of simplicity only two types of social security benefits have been considered here. Finally, it has been shown that vacancy chains instead of net employment growth should be taken into account. The main reason for this is that by means of a vacancy chain model net employment growth, migration, job mobility, and related income effects can be dealt with.

The model presented here is a theoretical extension of the presently operational demo-economic model of the Federation of Northern Economic Institutes (see F.N.E.I., 1977, 1978, 1981 and Oosterhaven, 1981). In a next stage some empirical results, especially with respect to the heavily underevaluated significance of social security benefits will be presented.

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